Edexcel IAL Physics A-level

Topic 1.3: Mechanics

Notes

1.3 - Mechanics

1.3.1 - Equations of motion

When an object is moving at **uniform acceleration**, you can use the following formulas:

$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t \qquad s = ut + \frac{at^2}{2} \qquad v^2 = u^2 + 2as$$

Where **s** = displacement, **u** = initial velocity, **v** = final velocity, **a** = acceleration, **t** = time.

When approaching questions which require the use of these formulas, it is useful to write out the values you know, and the ones you want to find out in order to more easily choose the correct formula to use.

For example:
A stone is **dropped** from a bridge 50 m above the water below. What will be its final velocity (v) and for how long does it fall (t)?

Note that in this example, the stone is dropped therefore we can assume initial velocity is zero. Also because the stone is dropped we know its acceleration will be g (9.81 m/s^2), which is the acceleration due to gravity.

s = 50 m **u = 0 m/s** **v = ?** **a = 9.81 m/s^2** **t = ?**

Using $v^2 = u^2 + 2as$, you can find v.

$v^2 = 0^2 + 2 \times 9.81 \times 50$ $\quad v^2 = 981$ $\quad v = 31.3$ m/s

Using $s = ut + \frac{at^2}{2}$, you can find t.

$50 = 4.905t^2$ $\quad t^2 = 10.19$ $\quad t = 3.19$ s

1.3.2-3 - Displacement, velocity and acceleration-time graphs and their properties

Distance - The distance travelled by an object is a scalar quantity and describes the amount of ground the object has covered.

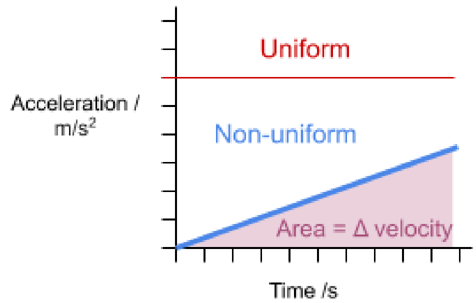
Displacement (s) - The overall distance travelled from the starting position (includes a direction and so it is a vector quantity).

Speed - This is a scalar quantity which describes the distance travelled per unit time.

Velocity (v) - rate of change of displacement - $\frac{\Delta s}{\Delta t}$

Acceleration (a) - rate of change of velocity - $\frac{\Delta v}{\Delta t}$

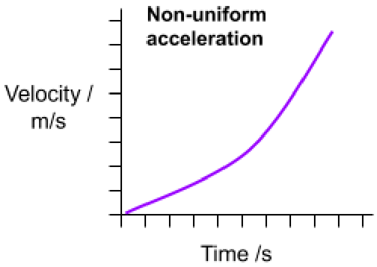
Uniform acceleration is where the acceleration of an object is constant.

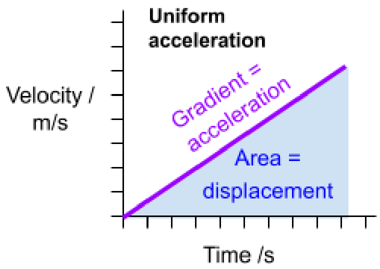


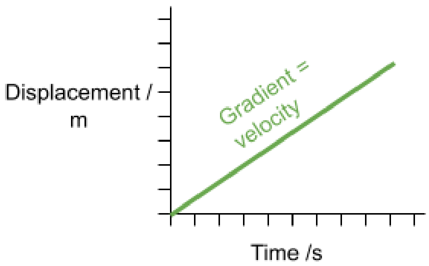
Acceleration-time graphs represent the change in velocity over time. Therefore, the **area** under the graph is the change in velocity.

Velocity-time graphs represent the change in velocity over time. Therefore, the **gradient** of a velocity-time graph is acceleration, and the **area** under the graph is displacement.

Displacement-time graphs show change in displacement over time, and so their **gradient** represents velocity.







Instantaneous velocity is the velocity of an object at a **specific** point in time. It can be found from a displacement-time graph by drawing a tangent to the graph at the specific time and calculating the gradient.

Average velocity is the velocity of an object **over a specified time frame**. It can be found by dividing the final displacement by the time taken.

1.3.4 - Scalars and vectors

Scalars and vectors are physical quantities, **scalars** describe only a magnitude while **vectors** describe magnitude and direction. Below are some examples:

Scalars	Vectors
Distance, speed, mass, temperature	Displacement, velocity, force/weight, acceleration

As vectors describe a magnitude and a direction, you **must always specify a direction** when giving a vector as a solution to a problem. There are many ways to do this, for example you may wish to give an absolute direction (e.g. North, East, etc), a direction relative to an object (e.g. Left, Right, etc) or give the angle that the vector makes with the horizontal (as defined by you or in the problem).

If a value is a vector named a, it may be represented as a bold letter (**a**), an underlined letter (<u>a</u>), or a letter with an arrow above it. The first representation mentioned is usually used when typing, while the next 2 are usually used in written algebra.

1.3.5 - Resolving vectors

Resolving a vector is where you split a vector into two parts which are perpendicular to each other, these are its **vertical** and **horizontal** components.

This can be done in two ways:

- **Calculation -**
  For this method, you use trigonometry to split a vector **V** into its components **x** and **y**, as shown below:

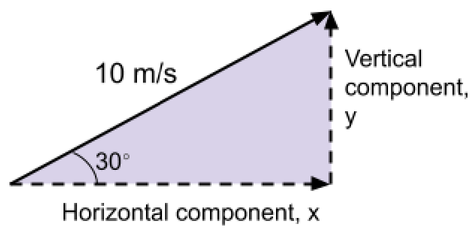
  $$x = V\cos\theta \qquad y = V\sin\theta$$

  However, if you struggle with remembering formulas a good hint to remember is:
  If you are moving from the original vector through the angle θ to get to your component (either horizontal or vertical), use cos.
  If you are moving away from the angle θ to get to your component, use sin.

  For example, a ball has been fired at a velocity of 10 m/s, at an angle of 30° from the horizontal, find the vertical and horizontal components of velocity.

  x= 10 cos 30°
  = 8.7 m/s

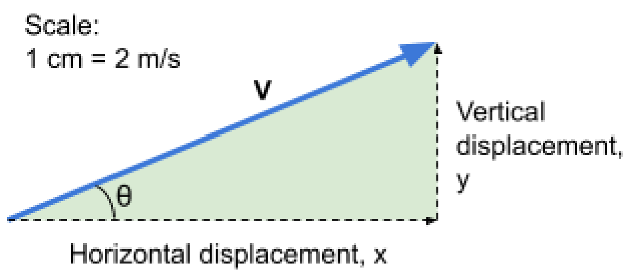
  y= 10 sin 30°
  = 5 m/s



- **Scale drawing -**
  First, you must choose an **appropriate scale** for your drawing and make sure to make a note of this somewhere on your page.

  Next, using a ruler and a protractor, draw the vector V (as described by the question). Then, draw the vector's horizontal and vertical components, making sure that they meet up at a right angle.

  Finally, measure the length of the components (that you have drawn), and find their value using the scale you used to initially draw V.



1.3.6 - Adding vectors

There are two methods you can use to add vectors:

- **Calculation -** this should be used when the two vectors are **perpendicular** to each other. In this method you use **Pythagoras' theorem** to find the magnitude of the vector and **trigonometry** to find its direction.

  For example, two forces are acting perpendicular to each other and have magnitudes of 5 N and 12 N. Find the resultant force, and its direction from the horizontal.

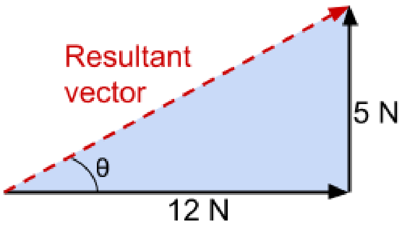
  To find the resultant magnitude (R) you can use **Pythagoras' theorem**:

  $12^2 + 5^2 = 169 = R^2 \qquad R = 13\ N$

  In order to find its direction, you can use **trigonometry**:

  $tan\ \theta = \frac{5}{12} \qquad \theta = 22.6°$

  Direction = 22.6° from the horizontal

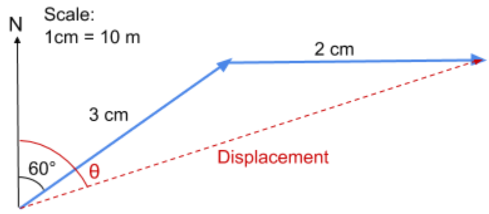


  It is very important to state **how** the angle you find signifies the direction.

- **Scale drawing -** this should be used when vectors are at **angles other than 90°.** For this method, you draw a **scale diagram** using a ruler and a protractor, in order to find the resultant vector. Make sure to note the scale that you are using in your diagram.

  For example, a ship travels 30 m at a bearing of 060°, then 20 m east. Find the magnitude and direction of its displacement from its starting position.

  Firstly, draw a **scale diagram**, using a ruler and a protractor as shown below, noting the scale you are using.



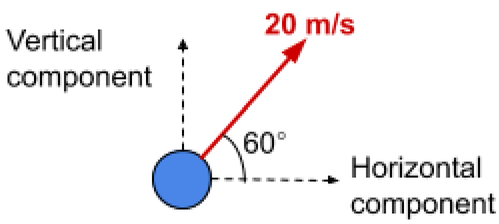
  Finally, measure the missing side and convert it to the magnitude using your scale and measure the missing angle θ, to find the bearing of the displacement.

  Magnitude = 4.9 cm = 49 m to scale   Direction = 072°

If two vectors are moving in the **exact** same direction, adding/subtracting them can be done by simply adding/subtracting their magnitudes. Otherwise, they will have to be resolved into their horizontal and vertical components, as explained earlier.

1.3.7 - Projectile motion

The vertical and horizontal components of a projectile's motion are **independent**, therefore the projectile's horizontal and vertical motion can be evaluated separately using the uniform acceleration formula, where acceleration is **constant**.



For example:
A ball is projected from the ground at 20 m/s, at an angle of 60° to the horizontal. Find the time taken for the ball to reach the ground and its maximum height. Ignore the effect of air resistance.

Firstly, you must resolve the initial speed into its components:

Vertical component = 20 sin 60° = 17.3 m/s

Horizontal component = 20 cos 60° = 10 m/s

(*To answer this particular question, you only need to consider the vertical component but this is not always the case*).

Maximum vertical height occurs when the vertical component of velocity first becomes 0, therefore:
s = ? u = 17.3 m/s v = 0 m/s a = -g (-9.81 m/s²) t = ?

Using $v^2 = u^2 + 2as$, you can find s.
$0 = 17.3^2 + 2 \times -9.81 \times s$ $19.62s = 300$ $s = 15.3$ m

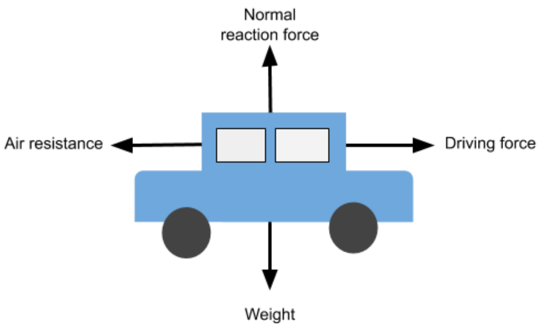
Maximum height = 15.3 m

Using $v = u + at$, you can find t.
$0 = 17.3 - 9.81t$ $9.81t = 17.3$ $t = 1.76$ s

Using the fact that the time for the journey will be double the time to reach the maximum height:
Time to reach ground = 3.5 s

1.3.8 - Free-body force diagrams

A **free-body diagram**, is a diagram which shows all the forces that act on an object, below is an example:



A free-body diagram will show you how each of the forces acting on the object **compare** with each other. In this example, all the arrows look equal, therefore we know that the car is travelling at a constant velocity (because the sum of the driving force and air resistance is zero - this is Newton's second law of motion and is discussed below).

1.3.9 - Newton's first and second laws of motion

Newton's 1st law - An object will remain at rest or travelling at a constant velocity, until it experiences a resultant force.

Newton's 2nd law - The acceleration of an object is proportional to the resultant force experienced by the object:

$$F = ma$$

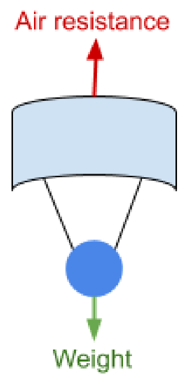
Where **F** is the resultant force (this could also be described as ΣF meaning the sum of all the forces acting on the object, which is equivalent), **m** is the object's mass and **a** is its acceleration.

Newton's second law can be used to find either resultant force, mass or acceleration, if two of the others are known, however, **the mass of the object must be constant**.

You can derive Newton's first law by considering Newton's second law, if you substitute in a **resultant force of 0 N**, you get an **acceleration of 0 m/s²**, which shows that the object is either at rest or travelling at a constant velocity.

Terminal velocity occurs where the frictional forces acting on an object and driving forces are equal, therefore there is no resultant force and so no acceleration, so the object travels at a constant velocity. A good example of an object reaching terminal velocity is a skydiver:

- As they leave the plane they accelerate because their weight (explained below) is greater than the air resistance acting on them.
- As the skydiver's speed increases, the magnitude of air resistance also increases. This continues until the force of weight and air resistance become equal, at which point **terminal velocity is reached**.



1.3.10 - Gravitational field strength and weight

Gravitational field strength (g) is the force per unit mass exerted by a gravitational field on an object. This can be calculated using the formula below:

$$g = \frac{F}{m}$$

Where **g** is the gravitational field strength, **F** is the force exerted and **m** is the mass of the object.

(Gravitational fields and gravitational field strength are covered in a lot more depth in the notes for Topic 12).

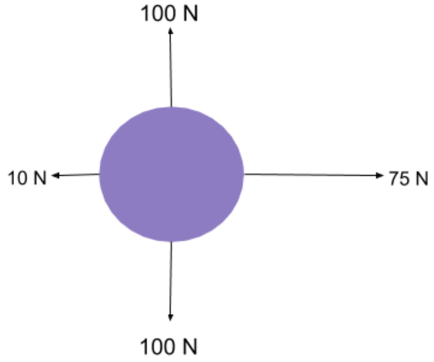
Weight (W) is the **gravitational force** that acts on an object due to its mass and is calculated by multiplying the object's mass by the gravitational field strength.

$$W = mg$$

Where **W** is the weight, **m** is the mass of the object and **g** is the gravitational field strength.

Below is an example where you would have to use the equation for weight above, Newton's 2nd law and a free-body diagram.

Find the acceleration of the ball in the diagram below:



Firstly, you must find the mass m, of the ball as you are only given its weight.

As **Weight = mass x g**, the mass = $\frac{100}{9.81}$ = 10.2 kg

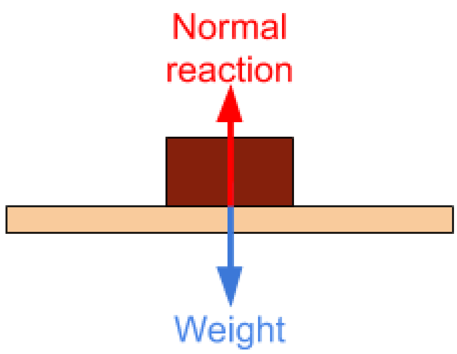
Next, you must find the resultant force (F).
$75 - 10 = 65$ N to the right

Finally, you can use **F = ma**, to find acceleration:
$65 = 10.2 \times a$ $\quad a = \frac{65}{10.2}$ $\quad a =$ **6.4 m/s^2**

2.20 - Newton's third law of motion

Newton's 3rd law - For each force experienced by an object, the object exerts an equal and opposite force.

As an example, consider a book lying on a table. The table experiences the force of the weight of the book and in return it exerts an **equal and opposite** normal **reaction** force, which means that the book will not experience a resultant force and so will remain stationary (by Newton's first law).



1.3.13 - Momentum

Momentum (p) is the product of the mass and velocity of an object.

$$Momentum = mass \times velocity$$

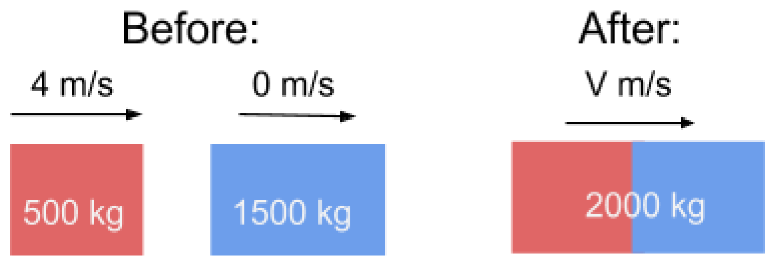
This equation can also be written as:

$$p = mv$$

1.3.14 - Principle of conservation of linear momentum

The **principle of conservation of linear momentum** is that momentum is **always conserved** in any interaction where **no external forces act**, which means the momentum before an event (e.g a collision) is equal to the momentum after.

This fact can be used to find the velocity of objects after collisions, for example:
A car with a mass of 500 kg, and a velocity of 4 m/s, collides with a stationary truck with a mass of 1500 kg. The two vehicles join together and move on with a velocity V. Find the value of V.



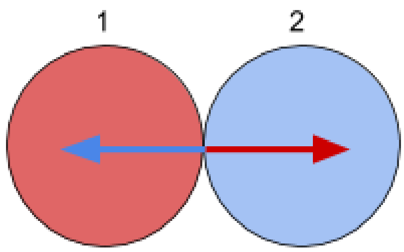
First find the momentum before the collision.
Total momentum before = $(500 \times 4) + (1500 \times 0)$ = 2000 kgm/s

Total momentum before = Total momentum after

Therefore, $2000 = 2000 \times V$ $V =$ **1 m/s**

Newton's second law states $F = ma$, therefore, $F = \frac{\Delta(mv)}{\Delta t}$ as $a = \frac{\Delta v}{\Delta t}$. From this you can see that force is the **rate of change of momentum**. *(This is discussed in more detail in topic 6).*

Using **Newton's third law**, you can see that the principle of conservation of linear momentum can also be expressed by saying that the **sum** of the change in momentum of the objects is **zero**. Newton's third law states: for each force experienced by an object, the object exerts an equal and opposite force. So, for two objects in a collision (labelled 1 and 2), the force exerted by 1 onto 2 is equal and opposite to the force exerted by 2 onto 1.



This can be written as:

$$F_1 = -F_2$$

Applying Newton's second law and $a = \frac{\Delta v}{\Delta t}$ (as above):

$$\frac{m_1 v_1}{\Delta t} = -\frac{m_2 v_2}{\Delta t}$$

Finally, you can multiply both sides by the change in time (as this will be equal in a collision between the two objects).

$$m_1 v_1 = -m_2 v_2$$
$$p_1 = -p_2$$
$$p_1 + p_2 = 0$$

1.3.15 - Moments

The **moment** of a force about a point is the force multiplied by the **perpendicular distance** from the **line of action of the force to the point**.

$$Moment = Force \times Perpendicular\ distance\ to\ the\ line\ of\ action\ of\ the\ force\ from\ the\ point$$

This can also be expressed as:

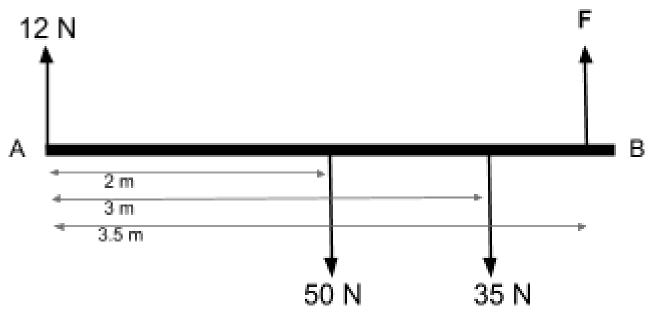
$$Moment = Fx$$

Where **x** is the **perpendicular** distance between the **line of action of the force and the axis of rotation**.

1.3.16 - Centre of gravity and the principle of moments

The **principle of moments** states that for an object in **equilibrium**, the **sum** of anticlockwise moments about a pivot is **equal** to the **sum** of clockwise moments.

You can use the principle of moments to help answer certain questions, for example:
Find the value of F from the diagram below.



Σ clockwise moments = Σ anticlockwise moments

Taking moments around A:

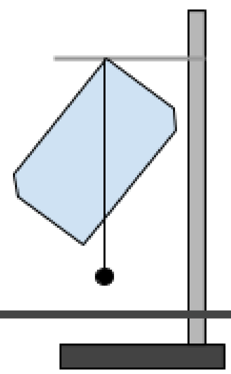
$$(2 \times 50) + (3 \times 35) = (3.5 \times F)$$

$$205 = 3.5F \qquad F = 58.6\ N$$

Note, in the example moments are taken about A, as the distance from A to A is 0, the moment caused by the 12 N force is also 0, therefore it can be ignored.

The **centre of gravity** of an object is the point at which gravity appears to act.
If an object is described as **uniform**, its centre of gravity will be exactly at its centre.
However, if an object is irregular (or not uniform), you can still find its centre of gravity by using the following method:

1. Attach the object and a plumbline (piece of string with a weight attached to it) to a clamp stand from the same point, so that the plumbline overlaps the object.
2. Wait until the object is stable, then draw a line on the object where the plumbline lies.



3. Repeat the above step at least 2 more times. The **intersection point** between all three lines is the **centre of gravity**.

As the centre of gravity is the point at which gravity appears to act, **the weight of an object acts at its centre of gravity**. It is vital to include the moment caused by the weight of an object, when calculating moments.

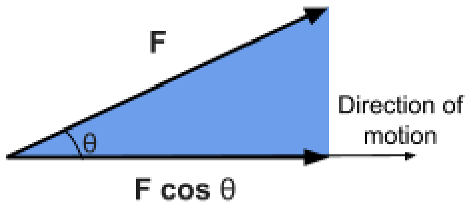
1.3.17 - Work

Work done (W) is defined as the **force causing a motion** multiplied by the distance travelled **in the direction of the motion**.

$$W = F\Delta s$$

Where **F** is the force along the direction of motion and **Δs** is distance travelled in the direction of the motion.

The force you are given in questions may not always be **along the direction of motion** (as defined in the formula above), in this case you will have to **resolve** the force as shown below.



In this case, the formula for work done is:

$$W = Fs\cos\theta$$

Where **s** is distance travelled and **θ** is the angle between the direction of the force and direction of motion.

1.3.18 - Kinetic energy

Kinetic energy (E_k) is the energy that an object has due to its motion. It can be calculated using the following formula:

$$E_k = \frac{1}{2}mv^2$$

Where **m** is the mass of the object and **v** is its velocity.

1.3.19 - Gravitational potential energy

Gravitational potential energy (E_{grav}) is the energy that an object has due to its position in a gravitational field. For example, on Earth, as an object is lifted up, the kinetic energy used to lift the object is converted into gravitational potential energy and stored by the object. The change in gravitational potential energy **near the Earth's surface** can be calculated using the formula below:

$$\Delta E_p = mg\Delta h$$

Where **m** is the mass of the object, **g** is the gravitational field strength and **Δh** is its change in height.

1.3.20 - Principle of conservation of energy

The **principle of conservation of energy** states that energy cannot be created or destroyed, but can be transferred from one form to another. Therefore, the total energy in a closed system stays constant.

$$Total\ energy\ in = Total\ energy\ out$$

As an example, think about a ball being thrown up into the air. The thrower gives the ball kinetic energy therefore it moves upwards, however as it does, the ball slows down because kinetic energy is transferred to gravitational potential energy. Eventually, all of the kinetic energy is transferred to gravitational potential energy and the ball stops momentarily, after which the ball's gravitational potential energy is converted back into kinetic energy and the ball falls to the ground.

It is very important to note that **work** is being done by the ball to work against resistive forces, therefore the initial kinetic energy given to the ball is not equal to the maximum gravitational potential when the ball has stopped in mid-air. This is because some of the kinetic energy given to the ball has been transferred to the environment in the form of heat due to air resistance.

Here is an example of a question where you could use the principle of conservation of energy, (noting that the effect of air resistance is ignored, therefore $\Delta E_p = \Delta E_k$):

As a simple pendulum of mass 500g swings, it rises up by a height of 10cm at its maximum amplitude from its equilibrium position. What is the maximum speed the pendulum can reach during its oscillation? (Ignore the effect of air resistance)

Firstly, find the maximum gravitational potential energy (which will be at the amplitude).
$\Delta E_P = 0.5 \times 9.81 \times 0.1$ **= 0.4905 J**

Then, equate this to the kinetic energy formula (subbing in known values), and rearrange to find v.
$\frac{1}{2} \times 0.5 \times v^2 = 0.4905$ $v^2 = 1.962$ $v =$ **1.4 m/s**

1.3.21 - Power

Power (P) is the **rate** of energy transfer. Power can be calculated by dividing the energy transferred or work done by the time taken, as shown below:

$$P = \frac{E}{t} \qquad P = \frac{W}{t}$$

Where **E** is the energy transferred, **W** is the work done, **t** is the time.

This is because work is a measure of energy transfer, and so the **rate** of doing work = the **rate** of energy transfer.

It is important to note that you can calculate the work done by an electrical appliance of power P in t seconds, by finding the product of power and time passed.

$$Energy\ transferred = P \times \Delta t$$

Below are some example questions using the formulas above:

163800 J of energy are required in order to boil 0.5 kg of water
How long will it take for a kettle with a power of 1200 W to boil 0.5 kg of water?

Power is the energy transferred per unit time, therefore to find the value of time taken we must divide the energy required by the power.

$P = \frac{W}{t} \rightarrow t = \frac{W}{P}$ $\qquad t = \frac{163800}{1200} =$ **136.5 s**

A ball of mass 0.6 kg is kicked and accelerates from rest to 12 ms^{-1} in 0.2 s. Calculate the average power gained by the ball.

Firstly, you must calculate the energy transferred, which in this case is the ball's gain in kinetic energy.

$E_k = \frac{1}{2}mv^2$ $\qquad \frac{1}{2} \times 0.6 \times 12^2 =$ **43.2 J**

Then, divide the energy transferred by the time taken.

$P = \frac{W}{t}$ $\qquad \frac{43.2}{0.2} =$ **216 W**

1.3.22 - Efficiency

Efficiency is a measure of how efficiently a system transfers energy. It is calculated by dividing the useful power output by total power input **or** by dividing the useful energy output by the total energy input.

$$Efficiency = \frac{useful\ power\ output}{total\ power\ input} \qquad Efficiency = \frac{useful\ energy\ output}{total\ energy\ input}$$